Technical Comments

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Comment on Special Section: New Perspectives on the Satellite Drag Environments of Earth, Mars, and Venus

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Nomenclature

A_p, $A_{//}$	=	flow projected area and flow parallel-surface area
C_D	=	drag coefficient based on flow projected area A_p
C_L	=	lift coefficient based on flow projected area A_p
c'	=	most probable thermal speed, m/s
n	=	atmospheric number density, $1/\mathrm{m}^3$
n'	=	molecular impingement rate per unit area and time, $1/(\mathrm{m}^2\ \mathrm{s})$
R	=	mean gas constant of atmosphere, $\mathrm{J}/(\mathrm{kg}\cdot\mathrm{K})$
S	=	molecular speed ratio V/c'
T_{atm}	=	static temperature of the undisturbed atmosphere $[p/(\rho R)]$, K
T_i	=	kinetic temperature of the approaching airstream $[V^2/(3R)]$, K
T_W	=	wall temperature, K
V	=	magnitude of freestream velocity (flight wind velocity), m/s
α_E	=	energy accommodation coefficient
θ	=	flow incidence angle of the surface panel (angle from surface normal to mass velocity vector of freestream)
σ	=	Maxwell accommodation coefficient

I. Introduction

THESE comments concern the contributions of Bruinsma and Forbes [1], Sutton et al. [2], and Tapley et al. [3] covering density and wind determination by the Challenging Minisatellite Payload (CHAMP) and Gravity Recovery and Climate Experiment (GRACE) satellites. The contributions use the accelerometer data of CHAMP and GRACE in combination with aerodynamic drag models to deduce atmospheric density, density variations, or atmospheric winds. In all three contributions and in many other publications on CHAMP (e.g., Bruinsma and Biancale [4]), the Cook model [5] is applied for the free-molecular aerodynamic force analysis. In their papers, the authors stress the importance of exact aerodynamic drag analysis for accurate density determination. In their error budget analysis, Bruinsma and Biancale [4] state an accuracy of the aerodynamic drag coefficient analysis between 5–10%, Tapley et al. [3] state an error of 10%, and Sutton et al. [2] state errors on the order of 5%. This Comment shows that, due to inappropriate use of the Cook model [5], the actual drag coefficients will be 30–100% higher, which directly influences the density determination.

II. Detailed Technical Comments

The authors use flat geometric macropanels to model CHAMP and GRACE for aerodynamic analysis. Figure 1 shows the geometry of both satellites. CHAMP is modeled with 13 panels and GRACE is modeled with 8 panels. For each wetted flat panel, a drag coefficient is calculated with Cook's method [5], as explained by Sutton et al. [2] [Eq. (6a)] and Bruinsma and Biancale [4] [Eq. (2)]. The exactly written Cook formula [5] is given as Eq. (1):

$$C_{Di} = 2\left[1 + \frac{2}{3}\sqrt{1 + \alpha_E\left(\frac{T_W}{T_i} - 1\right)}\cos(\theta)\right] \tag{1}$$

The equation is based on diffuse reflection with variable energy accommodation and molecular speed ratio $S = \infty$ (see Cook [5], Table 1, page 937). The second term in this equation gives the drag contribution of reflected molecules. There are two points to be cleared:

1) Misinterpretation of T_i as ambient atmospheric gas temperature T_{atm}. T_i originates from Schamberg's [6] gas–surface interaction model and was introduced by Cook [5] [Eq. (8)] as kinetic temperature T_i, which is related to incident kinetic energy.

2) Cook's formula [5] is not appropriate for geometries such as CHAMP and GRACE. Therefore, drag errors are much larger than stated in the commented papers.

A. Misinterpretation of T_i in Cook's Formula

The misinterpretation may result from the fact that the term *kinetic temperature* is often used in atmospheric physics to denote the temperature T in distinction with exospheric or molecular temperature T_M. The basic elements in the Cook formula [5] are as follows: A_{ref} is the reference area for C_{Di} (the flow projected panel area is used), $A_{\mathrm{ref}} = A_p$, α is the energy accommodation coefficient defined as

$$\alpha_E = \frac{E_i - E_r}{E_i - E_w} \tag{2}$$

T_W is the wall temperature, and T_i the kinetic temperature of incident particles with mass velocity V_i [Cook [5], Eq. 8].

Cook [5] replaced the energy ratio E_W/E_i by the equivalent temperature ratio T_W/T_i. For a particle gas with mass velocity V, the kinetic temperature T_i is given by $(3/2)RT_i = (1/2)V_i^2$. This therefore gives

$$T_i = \frac{1}{3R}V^2 \quad \text{or} \quad T_i = \frac{2}{3}S^2 T_{\mathrm{atm}} \tag{3}$$

We then obtain for C_D of Eq. (1),

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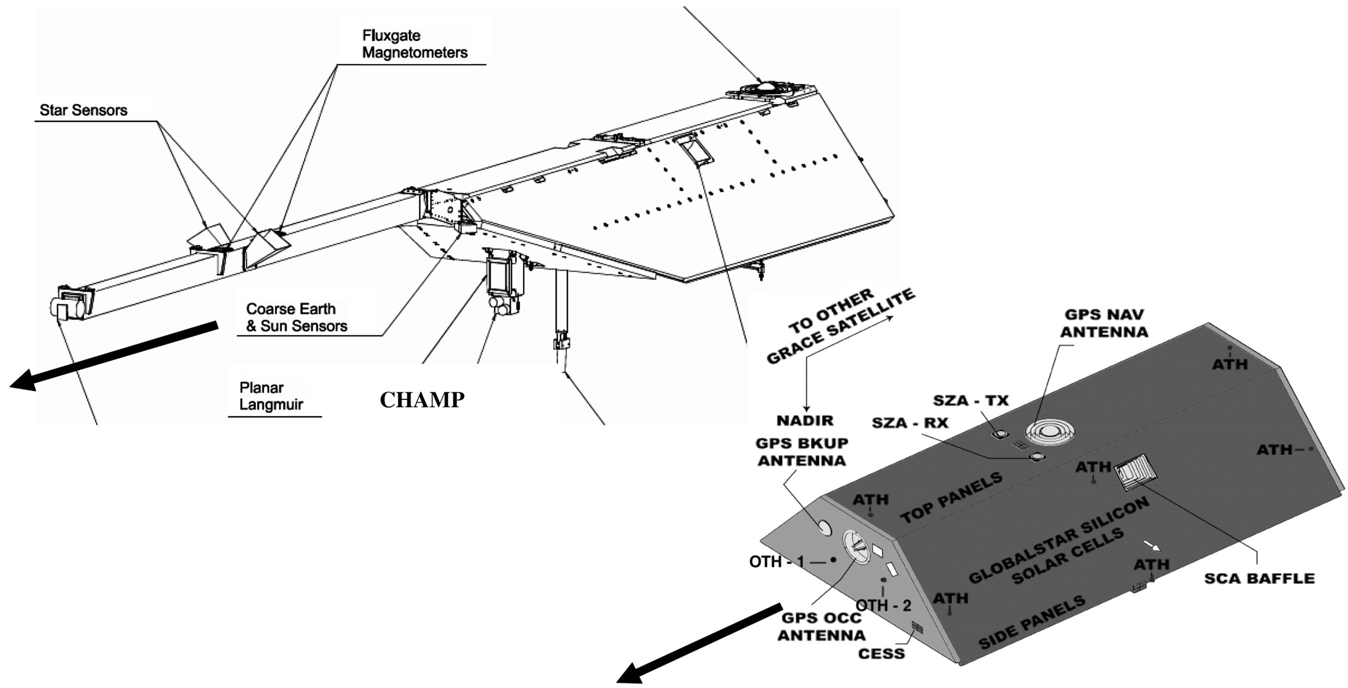

GRACE

Fig. 1 Geometry of CHAMP and GRACE.

$$
\begin{aligned}
C_{Di} &= 2\left[1 + \frac{2}{3}\sqrt{1 + \alpha\left(\frac{T_W}{T_i} - 1\right)}\cos(\theta)\right] \\
&= 2\left[1 + \frac{2}{3}\sqrt{1 + \alpha\left(\frac{T_W}{V^2/(3R)} - 1\right)}\cos(\theta)\right]
\end{aligned}
\tag{4}
$$

Evaluating T_i with $\boldsymbol{R} = 8314.3\ \mathrm{J/(kmol \cdot K)}$, $M = 16$ (kg/kmol), and $V = 7700$ m/s gives $T_i = 38,032$ K.

If we set for the temperature $T_W = 300$ K, the ratio T_W/T_i becomes extremely small: namely, $T_W/T_i = 0.008$. In the brackets of Eqs. (1) or (4), the value of $T_W/T_i = 0.008$ can be neglected against 1. For practical application, we obtain Eq. (5), which shows that for orbital conditions, the wall temperature influence vanishes in Cook's model [5]:

$$C_{Di} = 2[1 + \tfrac{2}{3}\sqrt{1-\alpha}\cos(\theta)] \tag{5}$$

This was explicitly stated by Cook [5] in his original paper on page 938, line 3. Thus, when using Cook's model, one should read his paper carefully. If one erroneously sets $T_i = T_{\mathrm{atm}}$, the ratios T_W/T_i have much higher values and the drag contribution of reflected particles will be overpredicted. The same applies for the lift formula used in the CHAMP and GRACE aerodynamic models:

$$
\begin{aligned}
C_{Li} &= \frac{4}{3}\sqrt{1 + \alpha\left(\frac{T_W}{T_i} - 1\right)}\sin(\theta) \\
&= \frac{4}{3}\sqrt{1 + \alpha\left(\frac{T_W}{V^2/(3R)} - 1\right)}\sin(\theta)
\end{aligned}
\tag{6}
$$

which can be written for $T_W \ll T_i$ as

$$C_{Li} = \tfrac{4}{3}\sqrt{1-\alpha}\sin(\theta) \tag{7}$$

As a demonstrative example in the following analysis, we used $T_W = 300$ K and $T_{\mathrm{atm}} = 1000$ K, which give $T_W/T_i = 0.3$. Figures 2 and 3 show the comparatively evaluated C_D and C_L values as functions of the energy accommodation coefficient for a flat plate normal to flow and for a plate with incidence. If one erroneously

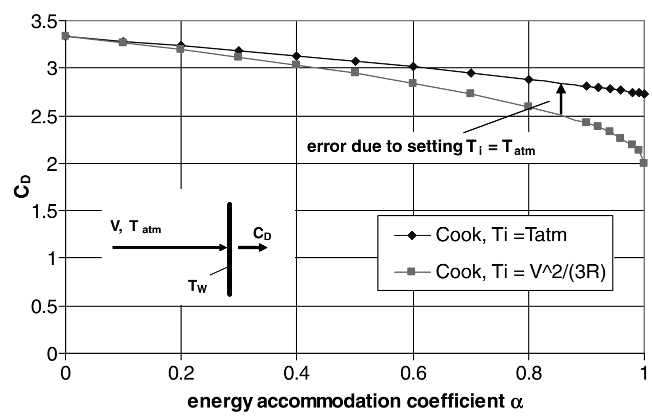

Fig. 2 Drag coefficient of a plate normal to flow as function of energy accommodation, Cook [5] model with $T_i = T_{\mathrm{atm}}$ and correct Cook values $T_W = 300$ K, $T_{\mathrm{atm}} = 1000$ K, and $V = 7700$ m/s.

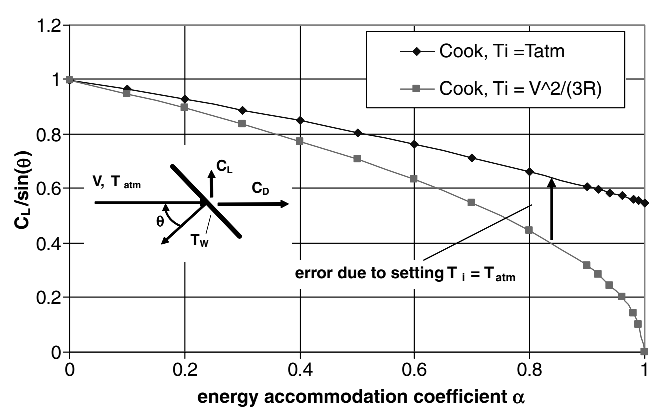

Fig. 3 Lift coefficient $C_L/\sin(\theta)$ as function of energy accommodation, Cook [5] model with $T_i = T_{\mathrm{atm}}$ and correct Cook [5] values $T_W = 300$ K, $T_{\mathrm{atm}} = 1000$ K, and $V = 7700$ m/s.

Table 1 Orbit data of CHAMP and GRACE and variation of atmospheric data, Committee on Space Research reference atmosphere 72

Quantity	CHAMP	GRACE
Orbit	Near-circular, polar	Near-circular polar
Mean altitude, km	450–300	500–485
Mean mol mass M, kg/kmol	16–17	15–16
Atmospheric temperature T, K	1800–600	1800–600
Mol. speed ratio $S = V/c'$	5.6–10	5.4–9.7
Nominal flight wind angles α and β	0	0
Flight velocity, m/s	7650	7650

sets $T_i = T_{\mathrm{atm}}$, a drag overprediction of 10–35% results for energy accommodation coefficients between 0.8 and 1; for lift, the overprediction is much larger. In hypersonic flow with $S \gg 1$, lift is only produced by reflected molecules. Thus, for $T_W \ll T_i$ and energy accommodation $\alpha = 1$, no lift can be produced. If we erroneously set $T_i = T_{\mathrm{atm}}$ at typical orbital conditions, we obtain T_W/T_{atm} values that may range between 0.2 and 0.6. Thus, the misinterpretation of Cook's [5] formula leads to an overprediction of lift, which can be more than 100%.

To avoid misinterpretations of T_i, it is recommended to replace T_W/T_i by its equivalent $T_W/T_i = 3RT_W/V^2$ in all Cook [5] formulas. Correct use of T_i can be found, for example, in the work of Moe and Moe [7] and in a recent attempt by Zuppardi [8] to implement the Schamberg [6] model in a direct simulation Monte Carlo code.

B. Limitations of Cook's Formula

Cook's formula [5] [Eq. (1)] has been derived for limiting hypersonic flow to inclined surface elements under the following conditions:

$$S = V/c' \gg 1 \quad \text{and} \quad Sn = S\cos\theta > 1 \tag{8}$$

where S is the molecular speed ratio, $c' = \sqrt{2RT_\infty}$ is the most probable thermal speed, and Sn is the normal component of S to surface panels. The formula neglects the influence of random molecular motion on aerodynamics. Frictional drag on surfaces parallel to flow is not predicted. It can only be applied under its derivation conditions (8).

The transformation equations of aerodynamic force coefficients C_{Di} and C_{Li} into accelerations neglect the possible drag of panels parallel to flow (Eq. 4 in [3] and Eq. 2 in [2]).

On a flat plate, parallel to the flow with area $A_{//}$, the molecular impingement rate of atmospheric particles due to the finite random thermal motion c is given by a formula independent on speed ratio S:

$$N_i' = \frac{n}{2\sqrt{\pi}} c' A_{//} \tag{9}$$

The particle impinging rate on the flow projected frontal area A_{pf} for $S^* \cos(\theta) \gg 1$ is given by

$$N_i' = nVA_{p,f} \tag{10}$$

Thus, the molecular impingement rate on flow parallel-surface panels starts to balance the impingement rate on the frontal area under the following condition:

$$A_{//}/A_{p,f} \geq 2\sqrt{\pi}S \tag{11}$$

At speed ratio $S = 7$, this is the case for $A_{//}/A_{p,f} = 24.8$

Each particle transports to the parallel-surface tangential momentum of m^*V. Thus, the drag coefficient due to molecules incident on flow parallel surfaces is given by

$$C_D = \frac{1}{\sqrt{\pi}} \frac{1}{S} \tag{12}$$

where S is the molecular speed ratio, and C_D is based on actual panel area $A_{//}$.

For diffuse reflection with variable energy accommodation coefficient α_E, the reemitted molecules will not contribute to the drag coefficient. If we use the simple Maxwell model for gas–surface interaction, only the fraction σ of the incident molecules, reflected diffusely, will contribute to the drag. The preceding drag formula reduces to

$$C_D = \frac{\sigma}{\sqrt{\pi}} \frac{1}{S} \tag{13}$$

III. Comparative Drag Coefficient Analysis of CHAMP and GRACE

CHAMP and GRACE orbit data and the atmospheric conditions are shown in the Table 1. It is evident that due to variation of atmospheric conditions, the freestream molecular speed ratio $S = V/c'$ may vary between $S = 5.4$ and 10. For an aerodynamic analysis, the shape of the spacecraft is of importance. The shape decides if a simple analysis based on hyperthermal free-molecular blunt-body flow can be applied or not. The shape analyses of CHAMP and GRACE shown in Fig. 1 lead to the following

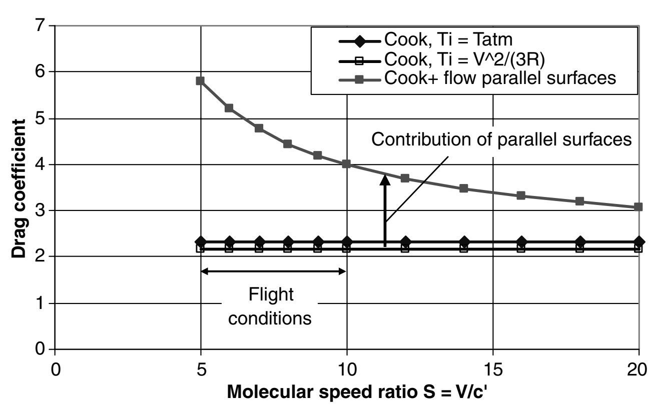

Fig. 4 Drag coefficient of CHAMP with Cook [5] model including flow parallel surfaces; energy accommodation coefficient $\alpha = 0.9$.

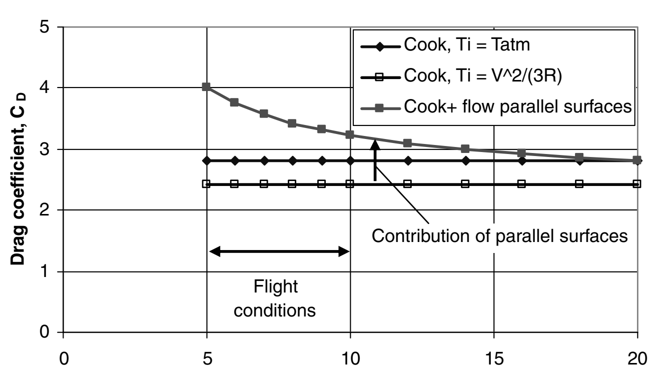

Fig. 5 Drag coefficient of GRACE with Cook [5] model including flow parallel surfaces; energy accommodation coefficient $\alpha = 0.9$.

Table 2 CHAMP and GRACE flow projected and flow parallel-surface areas

Spacecraft	Flow projected area A_p, m^2	Flow parallel-surface area $A_{//}$, m^2	Area ratio $A_{//}/A_p$
CHAMP	0.4701	16.78	35.70
GRACE	0.9556	15.01	15.71

conclusions: GRACE is in principle of convex shape; thus, on each surface panel, the incident fluxes can be calculated with exact gas-kinetic moment integrals. CHAMP has a long boom, which forms a concave compound with the oblique frontal area. Thus, the molecular incidence angle of particles on the frontal area is restricted and multiple wall collisions will occur. An analysis would require, as explained by Bird [9], a test particle Monte Carlo method, which is available in dedicated satellite aerodynamic codes such as RAMSES and ANGARA [10–13].

For simplicity, we use convex-body aerodynamic formulation. We compare an analysis based on Cook's [5] method with an analysis including the flow parallel surfaces. The necessary geometric data are taken from macromodel data given in [2] (Table 1) and [4] (Table 1). For normal flight conditions ($\alpha = \beta = 0$ deg), one obtains the data shown in Table 2 for flow projected area A_p and area A_t tangential to the flow.

Figure 4 shows the drag coefficient of CHAMP as a function of speed ratio S for 3 cases: case 1 is the Cook [5] model with an inappropriate setting of $T_i = T_{\mathrm{atm}}$. Case 2 is the Cook model with the correct setting of $T_i = V^2/(3R)$. Case 3 is the correct Cook model (case 2) plus the contribution of the parallel surfaces.

In the flight-condition range of $S = 5$–10, the viscous drag of parallel surfaces will, on average, double the drag. A similar analysis is shown in Fig. 5 for GRACE, which has less parallel surfaces and therefore the frictional-drag contribution is smaller. The frontal area is normal to flow; therefore, a reflected molecule with an energy accommodation of $\alpha = 0.9$ contributes more to the frontal area drag, as in the case of CHAMP. In the range of flight conditions of $S = 5$–10, the viscous drag of parallel surfaces will, on average, increase the total drag by approximately 30%.

IV. Conclusions

When using the Cook [5] model, the temperature ratio T_W/T_i should be used correctly. The Cook model applied to the CHAMP and GRACE spacecraft neglects the frictional-drag contribution of the large surfaces parallel to the flow. An analysis considering the skin friction on these flat panels gives drag coefficients, which are a minimum of 30% and a maximum of 100% higher. This has a direct impact on the atmospheric-density determination.

Satellites with accurate accelerometers (such as CHAMP and GRACE) and their follow-up generation [such as the Gravity Field and Steady-State Ocean Circulation Explorer (GOCE)] provide excellent possibilities to improve the thermosphere models. They can also be used to test and improve the model parameters for gas–surface interaction. The prerequisite is, however, a correct treatment of the free-molecular aerodynamics of these specially shaped satellites. For these tasks, one should rely on methods that are based on correct solutions of the free-molecular gas-kinetic equations.

References

[1] Bruinsma, S. L., and Forbes, J. M., "Storm-Time Equatorial Density Enhancements Observed by CHAMP and GRACE," *Journal of Spacecraft and Rockets*, Vol. 44, No. 6, 2007, pp. 1154–1159. doi:10.2514/1.28134

[2] Sutton, K. E., Nerem, J. M., and Forbes, J. M., "Density and Winds in the Thermosphere Deduced from Accelerometer Data," *Journal of Spacecraft and Rockets*, Vol. 44, No. 6, 2007, pp. 1210–1219. doi:10.2514/1.28641

[3] Tapley, B. D., Ries, J. C., Bettadpur, S., and Cheng, M., "Neutral Density Measurements from Gravity Recovery and Climate Experiment Accelerometers," *Journal of Spacecraft and Rockets*, Vol. 44, No. 6, 2007, pp. 1220–1225. doi:10.2514/1.28843

[4] Bruinsma, S., and Biancale, R., "Total Densities Derived from Accelerometer Data," *Journal of Spacecraft and Rockets*, Vol. 40, No. 2, 2003, pp. 230–236. doi:10.2514/2.3937

[5] Cook, G. E., "Satellite Drag Coefficients," *Planetary and Space Science*, Vol. 13, Oct. 1965, pp. 929–946.. doi:10.1016/0032-0633(65)90150-9

[6] Schamberg, R., "A New Analytical Representation of Surface Interaction for Hyperthermal Free Molecular Flow," Rand Corp. RM-2313, Santa Monica, CA, 1959.

[7] Moe, K., and Moe, M., "Simultaneous Analysis of Multi-Instrument Satellite Measurement of Atmospheric Density," *Journal of Spacecraft and Rockets*, Vol. 41, No. 5, 2004, pp. 849–853. doi:10.2514/1.2090

[8] Zuppardi, G., "DSMC Implementation of the Schamberg Model: Comparison with the Maxwell Model," *Rarefied Gas Dynamics*, American Inst. of Physics, CP 762, Melville, NY, 2005, pp. 632–637.

[9] Bird, G. A., *Molecular Gas Dynamics and the Direct Simulation of Gas Flows*, Clarendon, Oxford, 1994.

[10] Koppenwallner, G., Johannsmeier G., Klinkrad H., Ivanov M., and Kashkowski A., "A Rarefied Aerodynamics Modeling System for Earth Satellites, (RAMSES)," *Rarefied Gas Dynamics*, Vol. 2, edited by J. Harvey, G. Lord, Oxford Univ. Press, Oxford, 1995, pp. 1366–1372.

[11] Fritsche, B., Ivanov M., Kaskowski A., Koppenwallner G., Voskoboinikov U., and Zhukova G., "Radiation Pressure Forces on Complex Spacecraft," Hypersonic Technology Goettingen, CR ESOC/11908/96/D/IM, Katlenburg-Lindau, Germany, 1998.

[12] Fritsche, B., and Klinkrad, H., "Accurate Prediction of Non Gravitational Forces for Precise Orbit Prediction Part 1: Principles for the Computation of Force and Torque Coefficient," IAA/AAS Astrodynamics Specialist Conference, AIAA Paper 2004-2461, Providence, RI, 2004.

[13] Fritsche, B., and Klinkrad, H., "Accurate Prediction of Non Gravitational Forces for Precise Orbit Prediction Part 2: Determination of Perturbing Forces and Torques in Orbital Environment," IAA/AAS Astrodynamics Specialist Conference, AIAA Paper 2004-2462, Providence, RI, 2004.

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